

Moment of inertia

Solid sphere  $I = (2/5)MR^2$  & spherical shell  $I = (2/3)MR^2$

$$1. I_z = \int (r \sin \theta)^2 dm$$

$$2. I_z = \int (r \sin \theta)^2 \rho dV$$

$$3. I_z = \int (r \sin \theta)^2 \rho (r \sin \theta d\phi)(dr)(rd\theta)$$

$$4. I_z = \rho \int_{R_1}^{R_2} (r^4 dr) \int_0^{2\pi} d\phi \int_0^\pi \sin^3 \theta d\theta$$

$$5. I_z = \rho \left[ \frac{r^5}{5} \right]_{R_1}^{R_2} \left[ \phi \right]_0^{2\pi} \left[ \frac{4}{3} \right]$$

$$6. I_z = \rho \left[ \frac{R_2^5}{5} - \frac{R_1^5}{5} \right] [2\pi] \left[ \frac{4}{3} \right]$$

$$7. \text{FOR solid sphere; } R_1 = 0; R_2 = R; V = \frac{4}{3} \pi R^3$$

$$8. I_z = \frac{M}{\left(\frac{4}{3} \pi R^3\right)} \left[ \frac{R^5}{5} \right] [2\pi] \left[ \frac{4}{3} \right] = \frac{2}{5} MR^2 \quad \text{Q.E.D.1}$$

$$9. \text{FOR sphere\_Shell; } R_1 \approx R_2 \approx R; V = \frac{4}{3} \pi (R_2^3 - R_1^3)$$

$$10. V = \frac{4}{3} \pi (R_2^3 - R_1^3) = \frac{4}{3} \pi (R_2 - R_1)(R_2^2 + R_2 R_1 + R_1^2)$$

$$11. V = \frac{4}{3} \pi (R_2^3 - R_1^3) = \frac{4}{3} \pi (R_2 - R_1)(3R_2^2) = 4\pi (R_2 - R_1)(R_2^2)$$

$$12. I_z = \frac{8}{15} \pi \rho [R_2 - R_1] [R_2^4 + R_2^3 R_1 + R_2^2 R_1^2 + R_2 R_1^3 + R_1^4]$$

$$13. I_z = \frac{8}{15} \pi \frac{M}{4\pi (R_2 - R_1)(R_2^2)} [R_2 - R_1] [5R_2^4]$$

$$14. I_z = \frac{2}{3} MR^2 \quad \text{Q.E.D.}$$

